Technical Comments

Comment on "Dynamic Response of a Cylinder to a Side Pressure Pulse"

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THE response of a long circular elastic cylindrical shell to a cosine distributed impulsive load over half the shell circumference has recently been the subject of several analytical and experimental investigations. Traveling wave solutions for membrane stresses are presented in Refs. 1 and 2 and a modal solution for both membrane and bending stresses (based on the shell equations given in Ref. 3) is presented in Ref. 4. Unfortunately, the analysis of Ref. 4 is inaccurate in that critical terms were omitted from the frequency equation. In particular, the frequency associated with the fundamental bending mode is in considerable error. However, the solutions of Ref. 4 can easily be corrected by using the frequency equations derived in Refs. 5 and 6.

The analyses presented in Ref. 5 also suggest that a solution to the problem utilizing simplified shell equations which uncouple the extensional and inextensional vibrations should have the same range of validity as the coupled equations. Solutions for the membrane and bending stresses based on the purely extensional[1] and inextensional[7,8] equations of shell motion are

$$\frac{\sigma_m h}{Ic} = -\frac{1}{\pi}\sin\tau - \frac{1}{(2)(2)^{1/2}}\sin[(2)^{1/2}\tau]\cos\theta + \frac{2}{\pi}\sum_{n=2,4}^{\infty}\frac{(-1)^{n/2}}{(n^2-1)(n^2+1)^{1/2}}\sin[(n^2+1)^{1/2}\tau]\cos n\theta \tag{1}$$

$$\frac{\sigma_b h}{Ic} = \frac{2(3)^{1/2}}{\pi}\sum_{n=2,4}^{\infty}\frac{n(-1)^{n/2}}{(n^2-1)(n^2+1)^{1/2}}\cdot \sin\left[\frac{n(n^2-1)}{2(3)^{1/2}(n^2+1)^{1/2}}\cdot\frac{h\tau}{a}\right]\cos n\theta \tag{2}$$

$$\tau = ct/a, \quad c^2 = E/[\rho(1-\nu^2)] \tag{3}$$

where σ_m is the shell membrane stress, σ_b is the bending stress at the radially outward shell surface; h and a are the shell thickness and mean radius; E, ρ, and ν are Young's modulus, density and Poisson's ratio; t is time; I is the peak intensity of the impulsive load which is cosinusoidally distributed over $|\theta| < \pi/2$; and θ is the angular coordinate.

Stress responses predicted by Ref. 4 with the corrected frequency equation were compared with the stress responses predicted by Eqs. (1) and (2) at $\theta = 0, \pi/2, \pi$ and no differences could be detected in the plotted results.‡ However, Fig. 1 demonstrates the differences between the uncorrected and corrected results of Ref. 4.

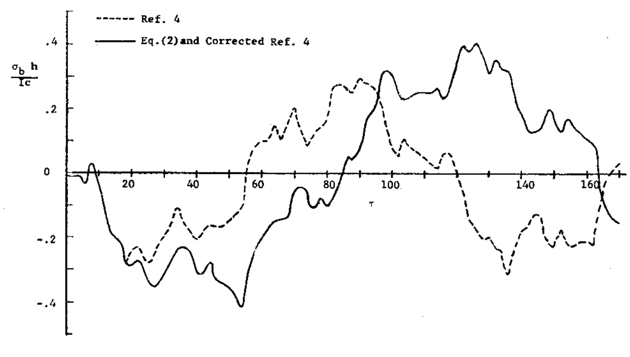

Fig. 1 Bending stress at $\theta = \pi$, $a/h = 20$.

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‡ Plots of these curves are available from the authors.

Reply by Author to M. J. Forrestal, M. J. Sagartz, and H. C. Walling

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FORRESTAL et al. claim that we used a modal solution, and that critical terms were omitted from the frequency equation. The modal method was not used for the solution, nor was any frequency equation presented, using the commonly accepted meaning of these terms. The solution was a series consisting of terms involving the products of a function of time and a function of the space variable, chosen to match the corresponding form of the loading function. The time-dependent portion of the solution

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was found directly, without first finding the normal modes of vibration. Therefore, the commentators' criticism of a "frequency" equation is not applicable. It is possible that Forrestal et al. are referring to some specific equation in the paper, but they did not indicate the equation or the missing terms to which they refer. Since the commentators do not give sufficient information to identify the object of their criticism in our solution, it is not possible to produce a specific reply. Nevertheless, this writer has re-examined the derivation of the differential equation governing the unknown time-dependent terms of the series solution, Eq. (17) in the paper, assuming that Forrestal et al. may be referring to it. This equation was found to be correct for the linear analysis of the problem examined.

The paper examined the broad problem of a nonaxisymmetric time-varying pressure pulse of finite duration applied to an infinitely long cylindrical shell using the linearized Flügge shell equations. Solutions were presented for several thickness-to-radius ratios, and for several types of loadings. One of these loadings was the limiting case of the "pure" impulse (zero duration time), with which Forrestal et al. have compared their modal solution. It must be noted that they show only the bending stresses in their figure, for an unspecified thickness-to-radius ratio. The peak bending stresses are always less than the peak membrane stresses for this problem, for thin shells, as indicated in the original paper, and thus the figure magnifies the importance of the differences between the two sets of results. They show an increase of 33% in the peak bending stress over our values, from $\sigma h/Ic = 0.3$ to 0.4. However, the peak membrane stress is about 0.7, as shown in our paper. If the total stress was plotted, it would show that peak values would increase from 1.0 to 1.1, or only 10%, using their results.

Furthermore, we stated in our paper that the purely elastic solution for the response would be useful only during early time periods, because the influence of damping or dissipative effects would become important soon after the load was cut off. Therefore, we did not consider that the response at large times, say $\tau > 20$, would be meaningful for a pure impulse load, since the peak stresses and deformations were surely to be attenuated. In this connection, it should be noted that Forrestal et al. show different results than ours only for those later times, when the usefullness of the solution is in doubt.

Comment on "Skin Friction on Porous Surfaces Calculated by a Simple Integral Method"

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ZIEN has recently described the application of a "refined" integral method[1] (essentially a moment-integral approach of the type originated by Tani[2] and subsequently developed by Lees and Reeves[3]) to obtain approximate laminar boundary-layer skin-friction solutions with surface blowing. The present Note is a commentary upon this work with a two-fold purpose: 1) comparison with the findings of a previous investigation[4] not cited by Zien which further illuminates the relative accuracy and utility of various approximate solution methods; 2) presentation of additional results for the blowing problem regarding the behavior of the displacement thickness.

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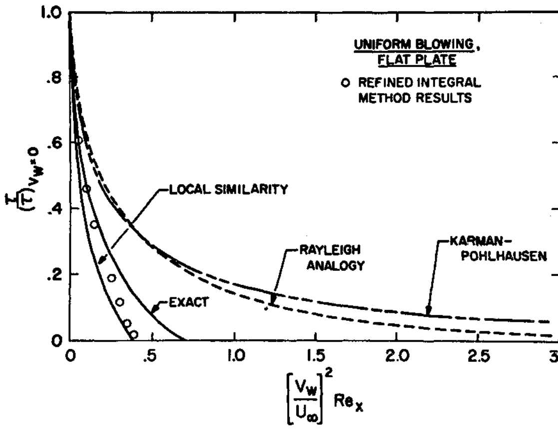

Fig. 1 Wall shear distribution.

In Ref. 4, skin-friction predictions were presented for non-similar flow along a flat plate with uniform blowing using a variety of approximate methods, including the Kârmân-Pohlhausen integral method with fourth-degree polynomial velocity profile, local similarity and the Rayleigh analogy. Figure 1 compares these findings with the newer results of Ref. 1 and the exact numerical solution of the problem.[5] Although this comparison further illustrates the general superiority of the refined integral method over the other approximations as regards skin friction, it is noteworthy that its accuracy does degrade considerably as the skin friction approaches zero (blow-off). Moreover, it is interesting to note that the well-known local similarity approximation also yields fairly good results in this example.

Whereas Zien presents results for skin friction only, it is also important to examine the corresponding blowing effect on displacement thickness growth, since this property is of great physical interest in the study of viscous-inviscid interaction[6] and boundary-layer separation[7] phenomena connected with the effects of surface mass transfer. This is particularly so in view of our previous findings[4] that approximations yielding satisfactory skin friction results do not necessarily do the same for the boundary-layer thickness parameters in the presence of blowing. Accordingly, we have calculated the $\delta^*(x)$ distribution pertaining to Zien's integral solution as well as to the various other aforementioned approximate solutions for uniform blowing; the results are compared with the exact behavior in

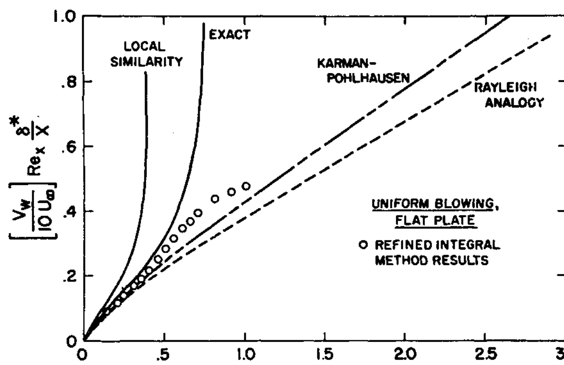

Fig. 2 Displacement thickness growth.